Homework8

Problem 1 Magnetic scalar "potential"

(a) Consider an infinite straight wire carrying current I. We know that the magnetic field outside the wire is $\vec{B} = (\mu_0 I/2\pi r)\hat{\theta}$. There are no currents outside the wire, so $\nabla \times \vec{B} = 0$; verify this by explicitly calculating the curl.

(b) Since $\nabla \times \vec{B} = 0$, we should be able to write $\vec{B}$ as the gradient of a function, $\vec{B} = \nabla\psi$. Find ψ, but then explain why the usefulness of ψ as a potential function is limited.

Problem 2 Field in the plane of a ring

A ring with radius R carries a current I. Show that the magnetic field due to the ring, at a point in the plane of the ring, a distance a from the center (either inside or outside the ring), is given by :

$$B = 2 \cdot \frac{\mu_0 I}{4\pi} \int_0^{\pi} \frac{(R - a\cos\theta)Rd\theta}{(a^2 + R^2 - 2aR\cos\theta)^{3/2}}$$

Hint: The easiest way to handle the cross product in the Biot-Savart law is to write the Cartesian coordinates of $\mathrm{d}\vec{l}$ and $\vec{r}$ in terms of an angle θ in the ring.

Problem 3 Copper solenoid

A solenoid is made by winding two layers of No. 14 copper wire on a cylindrical form $8\ \mathrm{cm}$ in diameter. There are four turns per centimeter in each layer, and the length of the solenoid is $32\ \mathrm{cm}$. From the wire tables we find that No. 14 copper wire, which has a diameter of $0.163\ \mathrm{cm}$, has a resistance of $0.010\,\Omega/\mathrm{m}$ at $75\,^\circ\mathrm{C}$. (The coil will run hot!) If the solenoid is connected to a $50\ \mathrm{V}$ generator, what will be the magnetic field strength at the center of the solenoid in gauss, and what is the power dissipation in watts?

Problem 4 A rotating solid cylinder

(a) A very long cylinder with radius R and uniform volume charge density ρ spins with frequency ω around its axis. What is the magnetic field at a point on the axis?

(b) How would your answer change if all the charge were concentrated on the surface?