**Lecture 1 Vector algebra**

1. **Vector algebra:**
* Scalar: Quantities has magnitude but no direction.

Mass (non-relativistic), charge, density, temperature,

* Vector: Quantities has magnitude and direction.

Displacement (), velocity (), acceleration ()

,

*
* (e.g )

**\*\*\*Kronecker symbol** :

 **\*\*\***symmetric

=(e.g. *,* )

**\*\*\*Levi-civita symbol**：

 **Determinant**

** **

1. **Position and displacement**
2. **Transformation of vector**
* is translated by , the resultant vector:

**Matrix method**:

* ****Rotation, is rotated around the -axis by angle ,

Matrix:

: rotation matrix

Generally for 3D rotation:

orthogonal matrix**:**  specific 3D rotation. **:** Transpose matrix

* Inversion
* Reflection with respect to xy, yz, and zx planes

xy-plane:

yz-plane:

zx-plane:

note: ,

 Inversion symmetry=

**All the transformations which keep unchanged can be decomposed into a series of the above operations.**

* **Concept: Pseudo-vector**

 electric field is vector, ordinary vector (polar vector), satisfy the above transformations.

 **pseudo vector** (axial vector).

**\*\*Under spatial rotation () , transforms as the same as .**

**\*\*Under inversion: , but**



 is even under spatial inversion, thus is quite different from ordinary vectors- pseudo-vector or axial vector. ( is also called polar vector).

**Note: Any quantity from cross product is a pseudo-vector (e.g., ) \*\*\*\*homework**

**\*\*\*\*\*Extended discussion (homework) how about reflection?**

****

**Lecture 2 Differential calculus**

1. **Gradient （梯度）**

Consider a scalar function

Where

, where is the angle between and .

Note: climb a hill , the **direction** of is along the steepest direction. The **magnitude** of is the slope along this steepest direction.



* Stationary point:

Saddle point: , maximum along one direction, minimum along another direction.

**Example：**

* Nabla (Del) operator: (vector operator)

The effect of is for any scalar function that:

1. **Divergence （散度）**

----applying the -operator on a vector function: point product.

For a vector field,

Each component is a function of ()

**Example: electric field:** )

 magnetic filed: )

**divergence:**

A scalar function has no divergence.

**Physical meaning**: is a measure of the vector **spreads out** from a point （a source point）.

**Example 1:**

consider a box, the net flow of to outside:

\*\*\*Think about electric current density.

**Example 2:** consider a small sphere with , the flux goes outside:

**Divergence: net flux per volume.** （）

1. **Curl （旋度）**

 is **a measure of how much “curls around” the point**.

The direction of is perpendicular to the circulation plane.

**Example:**

circulation around a circle of radial along the counter-clockwise direction.

**Circulation=**

（)

1. **Product rules (结合律)**

 : a constant

Linear operator properties

More

Not required to remember

1. **Laplacian: divergence of gradient**
* The curl of gradient is zero. **(梯度无旋)**

Similarly:

* The divergence of curl is zero. **（旋度无源）**
* The curl of curl.

 (leave for exercise)

definition:

**Lecture 3 Integral calculus**

**Fundamental law of calculus:**

1. **Line integral--path**

If the path forms a loop:

**Example:** The work done by following a path

**Generally speaking, this integral depends on the path.**

**For a special class of vector field, such an integral** is **path-independent,** which means that a close loop: .

For the force field satisfying this properties: **conservative force (保守力).**

e.g. **static gravity, electrostatic force**, etc.

**Example:**  to

Path 1:

Path 2:

 follows P1 and reverse along P2 coming back to a =>

1. **Surface integral**

, is an infinitesimal area, the direction is along the normal direction.

For a closed surface,

, the normal direction is from inside to outside.

If describe a flow of a liquid, then is the net flux.

**Example:**

Consider a cube with edge length **2**, the 5 surfaces (except for the bottom) form a big surface, calculate :

Set

Set

Set

 （set ）

 （set ）

1. **Volume integral**

 Where , is a scalar function.

Sometimes we also calculate volume integral of vectors:

**Example: ,**  for the prism

**Note: \*\*\***symmetric

\*\*\*Fully antisymmetric

**Lecture 4 Fundamental theorems of vector calculus**

1. **Fundamental theorems:** integral of a total derivative is determined by the boundary.
* **Calculus:**
* **Curved line**

Line integral boundary: two end

This integral is independent of the path from a to b.

: only for single valued function .

**if**  is a multi-valued function, say the azimuthal angle of the point , because is only uniquely defined up to then .

**In this class,** unless explicitly mentioned, we only consider single-valued function.

**Example:**

Following i+ii 🡪

Following iii, 🡪

For both cases

 **For any gradient field, the loop integral is zero.**

1. **Fundamental theorem of divergence-**

**\*\*\* Gauss theorem**

**within the volume = flow out through the surface.**

**Explanation:** let’s consider a small cube with center (x,y,z) and edge length , the flux pass the surface:

**Up & down:**

**Front &back Left & right:**

For a large volume, you can cut the system into a collection of small cubes.

****Then apply the above result and add them together.

, the sum of external surface, (all the contributions on internal surfaces cancel out.)

**Example** (see Lect. 3): with the cube, check the Gauss law:

Flux:

🡪🡪🡪

1. **Fundamental laws of curls – Stoke’s theorem**

Surface boundary

**Example:** Check for a planer version

**Note: only depend on the boundary, but not the surface.** A close area in 3D does not uniquely determine a surface. All the surface share the same boundary yield the same result.

For a close surface (e.g. a sphere):

Contract the boundary curve to a point.

 (a curl has no source)

**Example:** , consider square surface

1. **Integral by parts**

Note: In some situations, we can take the surface to infinity. If decays very quickly, the surface integral often vanishes.

**Lecture 5: curvilinear coordinates**

1. Cartesian coordinate

The grid of orthogonal:

Unit vectors: , ,

1. For many applications, we need curvilinear coordinates

**Example**: the surface of the earth

), the curves of surfaces

Say for the spherical coordinates: ),

, specifies a spherical surface.

, specifies a cone surface.

, specifies a half plane.

Unit vectors:

Transformations

Or from cartesian:

 SO(3), matrix

Det=1, S; special, O: orthogonal

Or:

**Lecture 6 -function and others**

1. **Definition: -function is a limit of distribution function**

Take the limit of , then doesn’t change.

But .

We define then

**Note:** there are many other ways to define through distribution functions.

**Example:** point charge at ,

the charge density:

1. principal value

Proof:

Properties

* , where is for the zero of

**Proof:** around each zero , Talor expansion

For any function

Set

Then:

Sum over all the zeros:

1. **3D- functions**
* **Example:**

For any

However , which independent on .

* This will be consistent if we set
* **Example**

: azimuthal angle, multi-valued

We know that at any point except , is regular 🡪  check?

🡪

🡪

* This will be consistent with

**Note:**