Homework10

Problem 1 Relativistic invariance

Prove that $\vec{E} \cdot \vec{B}$ and $E^2 - c^2 B^2$ are invariant under Lorentz tranformation of the electromagnetic field.

Problem 2 An electromagnetic wave

Here is a particular electromagnetic field in free space:

$$E_x = 0, \quad E_y = E_0 \sin(kx + \omega t), \quad E_z = 0;$$
$$B_x = 0, \quad B_y = 0, \quad B_z = -(E_0/c) \sin(kx + \omega t).$$

(a) Show that this field can satisfy Maxwell's equations if ω and k are related in a certain way.

(b) Suppose $\omega = 10^{10}\,\mathrm{s}{-1}$ and $E_0 = 1\,\mathrm{kV/m}$. What is the wavelength? What is the energy density in joules per cubic meter, averaged over a large region? From this calculate the power density, the energy flow in joules per square meter per second.

Problem 3 Traveling and standing waves

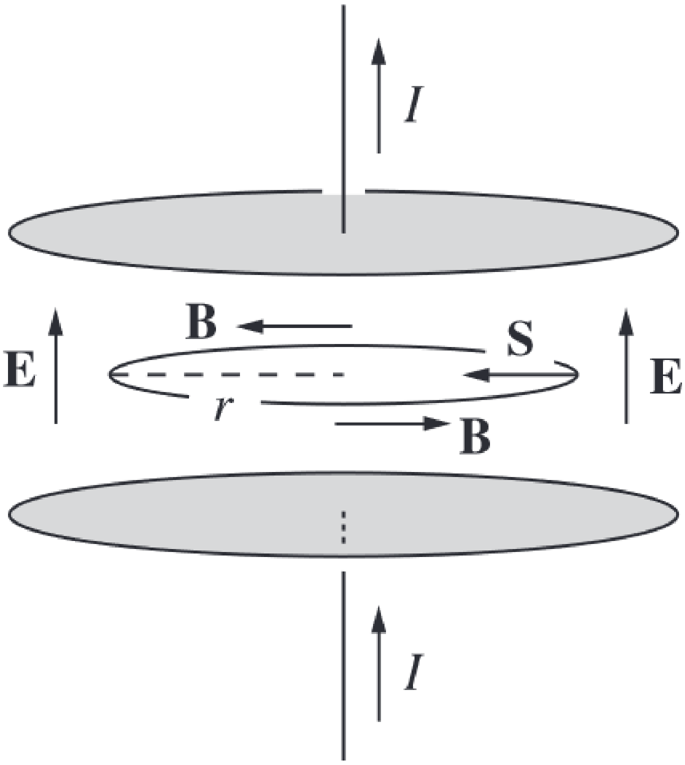
Consider the two oppositely traveling electric-field waves, $\vec{E}_1 = \hat{x} E_0 \cos(kz - \omega t)$ and $\vec{E}_2 = \hat{x} E_0 \cos(kz + \omega t)$. The sum of these two waves is the standing wave, $2\hat{x} E_0 \cos kz \cos \omega t$.

(a) Find the magnetic field associated with this standing electric wave by finding the $\vec{B}$ fields associated with each of the above traveling $\vec{E}$ fields, and then adding them.

(b) Find the magnetic field by instead using Maxwell's equations to find the $\vec{B}$ field associated with the standing electric wave, $2\hat{x} E_0 \cos kz \cos \omega t$.

Problem 4 Energy flow into a capacitor

A capacitor has circular plates with radius R and is being charged by a constant current I. The electric field E between the plates is increasing, so the energy density is also increasing. This implies that there must be a flow of energy into the capacitor. Calculate the Poynting vector at radius r inside the capacitor (in terms of r and E), and verify that its flux equals the rate of change of the energy stored in the region bounded by radius r.



Problem 5 Field momentum of a moving charge

Consider a charged particle in the shape of a small spherical shell with radius a and charge q. It moves with a nonrelativistic speed $\vec{v}$. The electric field due to the shell is essentially given by the simple Coulomb field, and the magnetic field is then given by $\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}$. Using the results from Lec. Note 15 section 4 , integrate the momentum density over all space. Show that the resulting total momentum of the electromagnetic field can be written as $m\vec{v}$, where $m \equiv (4/3)(q^2/8\pi\epsilon_0 a)/c^2$.