Homework5

Problem 1

A tetrahedron has equal resistors R along each of its six edges. Find the equivalent resistance between any two vertices. Do this by:
(a) using the symmetry of the tetrahedron to reduce it to an equivalent resistor;
(b) laying the tetrahedron flat on a table, hooking up a battery with an emf $\mathcal{E}$ to two vertices, and writing down the four loop equations. It's easy enough to solve this system of equations by hand, but it's even easier if you use a computer.

Problem 2

A capacitor initially has charge Q. It is then discharged by closing the switch in the Fig.1a below. You might argue that no charge should actually flow in the wire, because the electric field is essentially zero just outside the capacitor; so the charges on the plates feel essentially no force pushing them off the plates onto the wire. Why, then, does the capacitor discharge? State (and justify quantitatively) which parts of the circuits are the (more) relevant ones in the two setups (b and c).

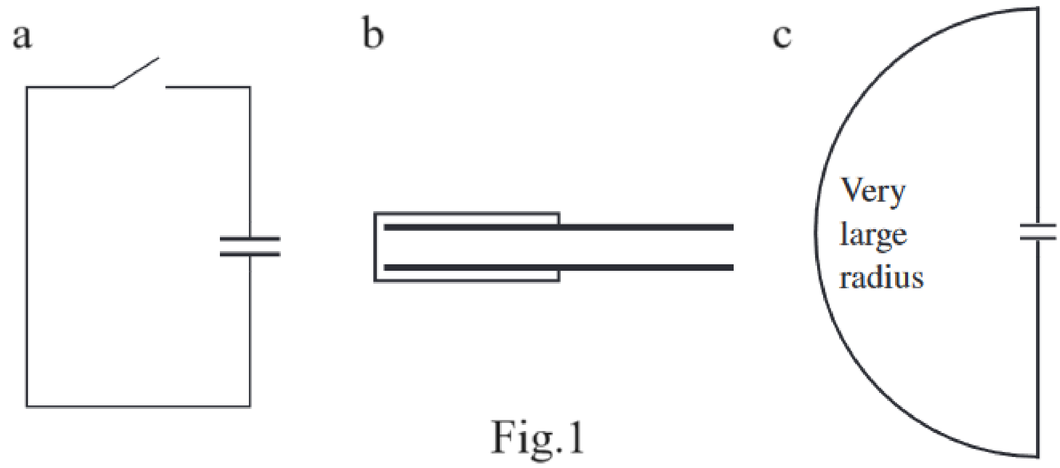

Fig.1

Problem 3

A discharge with two capacitors
(a) The circuit in the figure below contains two identical capacitors and two identical resistors. Initially, the left capacitor has charge Q_0 (with the left plate positive), and the right capacitor is uncharged. If the switch is closed at $t = 0$, find the charges on the capacitors as functions of time. Your loop equations

should be simple ones.

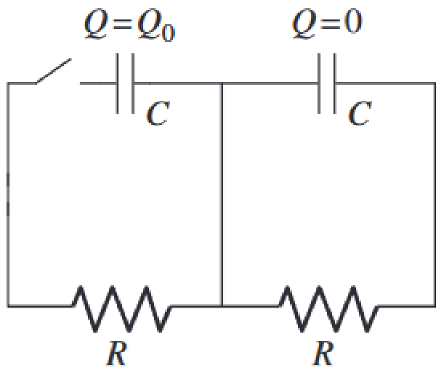

Fig.2

(b) Answer the same question for the circuit in Fig.3 , in which we have added one more (identical) resistor. What is the maximum (or minimum) charge that the right capacitor achieves? Note: Your loop equations should now be more interesting. Perhaps the easiest way to solve them is to take their sum and difference. This allows you to solve for the sum and difference of the charges, from which you can obtain each charge individually.

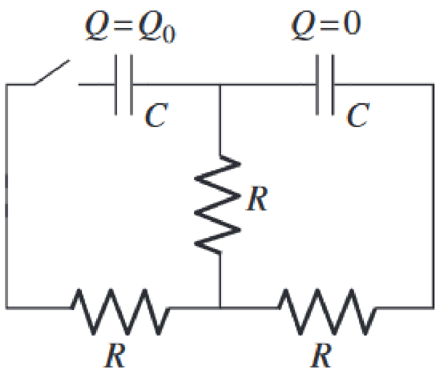

Fig.3

Problem 4 Space-time derivatives

Consider the Lorentz transform between two frames F and F' in the case that F' is moving at the velocity u along the x-direction with respect to F , i.e.,

$$\begin{pmatrix} x'^{,1} \\ x'^{,0} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x^1 \\ x^0 \end{pmatrix}$$

where$x^0 = ct$, $\beta = u/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. Prove that the derivatives defined in the following way $\partial^1 = \frac{\partial}{\partial x}$, and $\partial^0 = -\frac{1}{c}\frac{\partial}{\partial t}$ also satisfy the above Lorentz transforamtion.

Problem 5 Relatidstic doppler shift.

Protons are accelerated through a potential of $20\ \mathrm{kV}$, after which they drift with constant velocity through a region where neutralization to H atoms and associated light emission takes place. The H_β emission ($\lambda = 4861.33\ \mathring{\mathrm{A}}$ for an atom at rest) is observed in a spectrometer. The optical axis of the spectrometer is parallel to the motion of the ions. The spectrum is doppler-shifted because of the motion of the ions in the direction of observed emission. The apparatus also contains a mirror which is placed so as to allow superposition of the spectrum of light emitted in the reverse direction. Recall that $1\mathring{\mathrm{A}} \equiv 10^{-10}\ \mathrm{m}$.

(a) What is the velocity of the protons after acceleration?

(b) Calculate the first-order doppler shifts, depending on v/c, appropriate to the forward and backward directions, and indicate the appearance of the relevant part of the spectrum on adiagram.

(c) Now consider the second-order, or v^2/c^2, effect which arises from relativistic considerations. Show that the second-order shift is $= \frac{1}{2}\lambda(v^2/c^2)$, and evaluate this numerically for this problem. Notice that it is the same for both $+\vec{v}$ and $-\vec{v}$ motions.