Homework7

Problem 1

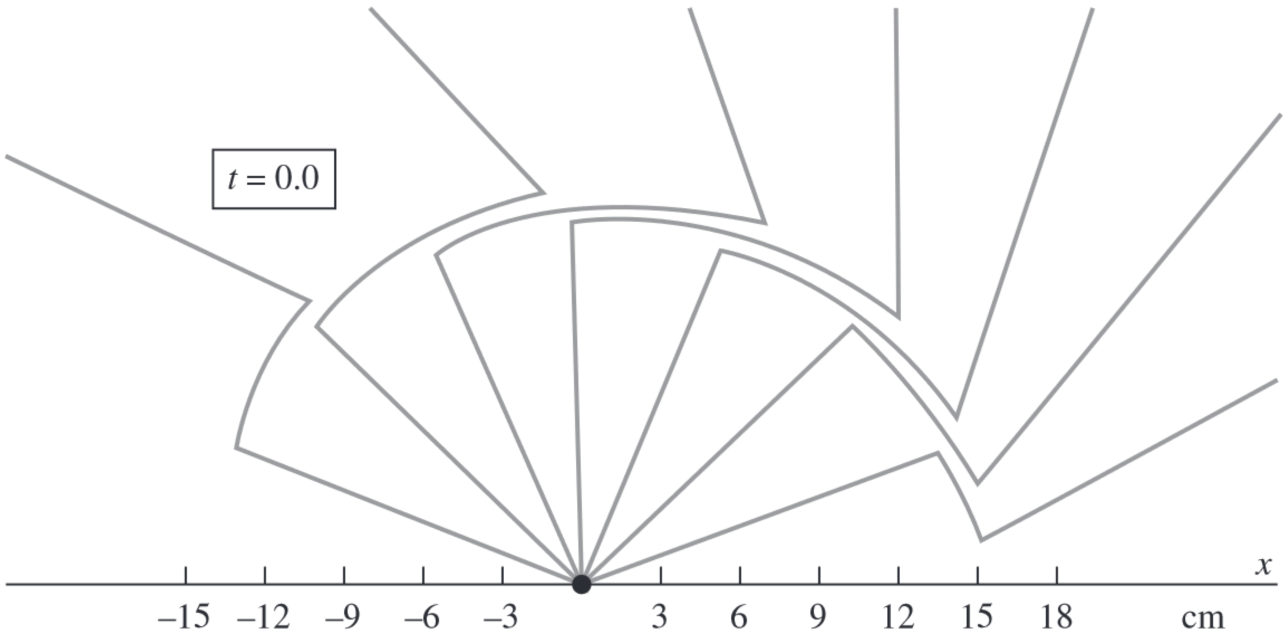
Inverse Compton effect. Work out the exact formula for the wavelength of a photon of wavelength λ scattered straight backward by an electron of velocity βc .
Apply to the case in which the incident photon has energy $(hv) = 3.0\mathrm{eV}$ and the electron has total energy $1.02\mathrm{MeV}$ $(\gamma = 2)$ to find the energy of the scattered photon.

Problem 2

In the field of the moving charge Q, given by $E' = \frac{Q}{4\pi\epsilon_0 r'^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}$, we want to find an angle δ such that half of the total flux from Q is contained between the two conical surfaces $\theta' = \pi/2+\delta$ and $\theta' = \pi/2-\delta$. You should find that, for $\gamma \gg 1$, the angle between the two cones is roughly $1/\gamma$.

Problem 3

In the figure you see an electron at time $t = 0.0$ and the associated electric field at that instant. Distances in centimeters are given. in the diagram.



(a) Describe what has been going on. Make your description as complete and quantitative as you can.
(b) Where was the electron at the time $t = -7.5\times 10^{-10}$ s?
(c) What was the strength of the electric field at the origin at that instant?

Problem 4

Two protons are moving parallel to one another a distance r apart, with the same velocity βc in the lab frame. According to $E' = \frac{Q}{4\pi\epsilon_0 r'^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}$, at the instantaneous position of one of the protons the electric field E caused by the other, as measured in the lab frame, is $\gamma e/4\pi\epsilon_0 r^2$. But the force on the proton measured in the lab frame is *not* $\gamma e^2/4\pi\epsilon_0 r^2$. Verify that by finding the force in the proton rest frame and transforming that force back to the lab frame. Show that the discrepancy can be accounted for if there is a magnetic field β/c times as strong as the electric field, accompanying this proton as it travels through the lab frame.

Problem 5

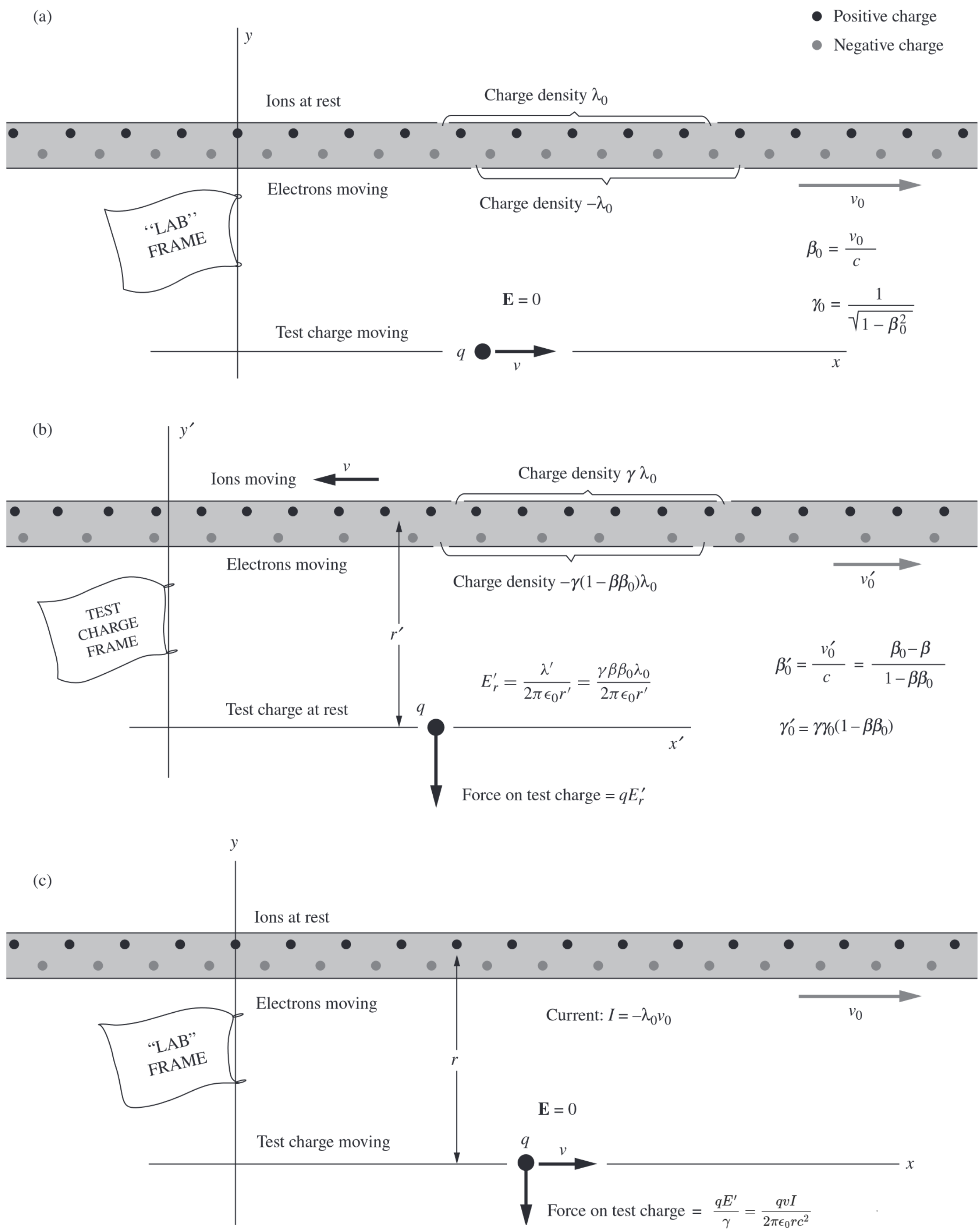
Consider a composite line charge consisting of several kinds of carriers, each with its own velocity. For one kind, k, the linear density of charge measured in frame F is λ_κ and the velocity is $\beta_k c$ parallel to the line. The contribution of these carriers to the current in F is then $I_k = \lambda_k \beta_k c$. How much do these k-type carriers contribute to the charge and current in a frame F' which is moving parallel to the line at velocity $-\beta c$ with respect to F ? By following the steps we took in the transformations in Figure below (next page), you should be able to show that

$$\lambda'_k = \gamma\left(\lambda_k + \frac{\beta I_k}{c}\right), \quad I'_k = \gamma(I_k + \beta c \lambda_k)$$

If each component of the linear charge density and current transforms in this way, then so must the total λ and I:

$$\lambda' = \gamma\left(\lambda + \frac{\beta I}{c}\right) \quad I' = \gamma(I + \beta c \lambda)$$

You have now derived the Lorentz transformation to a parallel-moving frame for any line charge and current, whatever its composition.


(a)
Positive charge
Negative charge
y
Ions at rest
Charge density λ_0
Electrons moving
Charge density $-\lambda_0$
v_0
"LAB" FRAME
$\beta_0 = \frac{v_0}{c}$
$\mathbf{E} = 0$
$\gamma_0 = \frac{1}{\sqrt{1-\beta_0^2}}$
Test charge moving
q
v
x
(b)
y'
Ions moving
v
Charge density $\gamma\lambda_0$
Electrons moving
Charge density $-\gamma(1-\beta\beta_0)\lambda_0$
v_0'
TEST CHARGE FRAME
r'
$E_r' = \frac{\lambda'}{2\pi\epsilon_0 r'} = \frac{\gamma\beta\beta_0\lambda_0}{2\pi\epsilon_0 r'}$
$\beta_0' = \frac{v_0'}{c} = \frac{\beta_0-\beta}{1-\beta\beta_0}$
Test charge at rest
q
x'
$\gamma_0' = \gamma\gamma_0(1-\beta\beta_0)$
Force on test charge $= qE_r'$
(c)
y
Ions at rest
Electrons moving
Current: $I = -\lambda_0 v_0$
v_0
"LAB" FRAME
r
$\mathbf{E} = 0$
Test charge moving
q
v
x
Force on test charge $= \frac{qE'}{\gamma} = \frac{qvI}{2\pi\epsilon_0 rc^2}$