**Lecture 10 Magnetostatics**

1. **Ampere’s law**

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We start from this formula to derive Biot-Savart’s law.

 Stoke’s theorem

 ---- ( ---- )

Clearly Ampere’s law only applies to **steady current**:

 ⟹

How about the divergence of . The fact is that so far no magnetic monopole is discovered. For any closed surface, the mangeic flux is zero

 Magnetic field has no source.

1. **Vector potentia**

 ⟹ , is well defined up to .

⟹

We further impose the condition (Coulomb gauge):

⟹

Assuming goes to zero at infinity, we can read off from the solution of Poisson equation:

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Check

Assuming at boundary.

**Example**: 2D, duality

Assume an 2D distribution , which does not depend on . We cannot use the above formula, because extends to .

 only has in-plane component, define and

⟹

 ⟹

 (Compare with ---- Gauss equation)

(, Lec\*. 2 )

So we transform it to an electro-static problem. Say

⟹ ⟹

⟹

short range cutoff

1. **Magnetic fields from a general shape of wire**

 cross section of wire

⟹ =(

⟹ Biot-Savart law

The contribution to from this line segment is

 is constant vector with respect to

⟹ ( Lec\* 2 )

⟹ , where

⟹ = Biot-Savart law

1. **Magneto-static boundary condition**

When there is surface current , magnetic fields can be discontinuous. But the **normal component remains continuous:**

 ⟹

⟹



 surface current density

⟹ = cross configuration

For arbitrary loop and field direction:

⟹

(vector algebra: see Lec\* 1)

* We may also need **boundary conditions for vector potential** . Since satisfies 2nd-order differential equation,  **itself is continuous** and the discontinuity may appear at its derivative.

 ⟹ the normal component of is continuous.

 as the thickness of The parallel component of is continuous.

Set the frame and

Only this component of is discontinuous. We don’t expect discontinuity of , because is continuous and is parallel to the boundary. The discontinuity comes from .

Or

**In comparison:** boundary conditions for electric surface charge

1. **Multiple expansion of vector potential**

We need to use

: Legendre polynomials (see the end)

**monopole dipole quadrupole**

* **Magnetic dipole component**

=

Using the identity , is a constant vector and .

⟹ where **magnetic moment**

 is the area of a surface, which is enclosed by the boundary (c.f. Prob. 1.61).

1. **Field from a dipole**

(Using Lec\*. 3)

⟹ Lec\*6.

Neglected only singular point at origin

Given ()

⟹

⟹

 see notebook

****⟹

⟹

* **Interaction between two magnetic dipoles**

**Magnetic energy** of a moment under external field:

Interaction between two dipoles:

=

\*\*\*Legendre polynomials：

Definition:

, ,