**Lecture 11 examples of magnetic fields**

1. **Cycloid motion of charge under** Griffiths p213, Ex. 5.2:

Related to the classic picture of quantum Hall edge.

**Recall**: equilibrium case of a charge:



* **Cyclotron motion** of a charge under without :

 （centripetal force 向心力）

⟹ cyclotron radius and frequency .

For the existence of , **spiral motion**

* **General case: cycloid motion** of charge under

Solution: no force in the x-direction. The motion is in the yz-plane.

 ,  *(see )*

 define cyclotron frequency

 ⟹

*(comparison: with harmonic function)*

⟹

⟹

⟹

⟹

⟹

Given the initial condition:

⟹ ⟹

⟹

Define： radius

⟹

 Cycloid velcoity

1. **Start from Biot-Savart law to prove** .

*(See )*

*(See and )*

-

*(See )*

The second term

*(See )*

The second term vanishes after volume integral.

*(See )*

1. **Application of Biot-Savart law**
2. **B-field from a long straight line:**

 point out of page, with the magnitude

 ----Ampere’s law

**The force between two parallel wires**

 line charge density

1. **B-field at distance above** the center of a circular loop of radius with a steady current

 has the magnitude of with the direction of a polar angle with respect to the -axis as plotted.

⟹

 or

Other components average to zero owning to rotation symmetry.

1. **B-field at axis of a wound solenoid**

Suppose that the left and right ends span the polar angles of and , respectively.

 ()

where and is the number of turns per length.

Suppose for an **infinitely long** solenoid,

, ⟹



**-field of a solenoid**

1. **Application of Ampere’s law + symmetric analysis (Other than Biot-Savart law)**
* **can only be along “circumferential” direction.**

 ⟹ ⟹

* **from a sheet current**

 should have **translational symmetry**, i.e. is uniform along xy-direction.

1. **Q:** Can has a z-component?

**** **A**: No. The system has the symmetry of combine time-reversal and rotation along z-axis by . This operation flips the direction of . So only have the in-plane component.

1. The system has the symmetry of **combined time-reversal and reflectional symmetry** with respect to zy-plane.

 is an axial (pseudo) vector ⟹

1. can only be along y-direction.

Let us choose a loop at ⟹

 should not depend on for .

1. For a loop crossing the current sheet.

The system has rotation symmetry around x-axis by .

⟹ .

⟹ ⟹

* **-field from an infinitely long solenoid**

The system has **rotational symmetry** around the axis.

1. cannot have radial component, otherwise .
2. cannot have “circumferential” component, otherwise .
3. can only be along the axial. It can also be proved that is uniform inside the solenoid, and outside.
4. If we set (limit case), ⟹

 along axial axis. : the number of turns per unit length.

* **A toroidal coil of a circular ring. The winding is uniform. What’s the distribution of -field? There are two ways:**
1. **Application of Biot-Savart law**

Our system has rotational symmetry around -axis, without loss of generality, let us consider a point in the xz-plan with .

The coordinate of on the toroidal coil: azimuthal angle.

The current density has no dependence.

 *Integral over*

The contribution to and are odd function of ⟹ vanishes after integration.

⟹ only along the -direction thus .

Similar calculation for the point yz-plane, is along -direction.

These imply that  **is a “circumferential” field.**

1. **Application of symmetry analysis.**

 is an axial-vector. It has different properties under reflection operation.

Can you explain why?

1. Our system has reflection symmetry respect to any vertical-radial plane.

⟹ cannot be parallel to that plane. can only be perpendicular to the vertical -radial plane.

⟹ is circumferential.

Then the results are straightforward. For point inside the torus with radius to the z-axis: , : total turns, otherwise .

* **-field of a rotation spherical shell**

The charged shell circulates around z-axis, let’s calculate at . is at xz-plane with with respect to z-axis



 rotate

🡪 🡪

To calculate, we first rotate to the z-axis and to the xz plane.

 is on the sphere with (spherical coordinate)

where surface current density,

Here: ,

Those terms contain goes to zero after average over

⟹ only . along = the direction of .

One have

🡪

Remember :

This expression is independent on the chosen of frame.

If set back along z-axis

⟹ along the azimuthal direction.

* i. For ,

*(see , )*

⟹

1. For ,

(see Lec\*. 6 and Lec. 10)

Since , because .

 (,)

 ---- SI-unit

Compare with dipole field

We can derive the magnetic moment:

Or and ---- CGS-unit