**Lecture-16 Conservation law II**

Page 359 Griffiths’ book

1. **Conservation of momentum**

**Example:** Calculate the net force on the “northern” hemisphere of a uniformly charged solid sphere of radius and charge .

**Solution:**

* **Method 1** using the stress tensor

 (in our case 🡪 and only contains E-field)

🡪

🡪

The boundary surface consists of two part- a hemisphere “bowl” at radius and a circular disk of the cross section at xy-plane.

1. **Total force on the bowl**

,

The total force is along z-axis due to the rotational symmetry along the z-axis.

🡪

Force on the “bowl”:

1. **Force on the disc:**

,

Force on the disc:

**Total force**

* **Method 2:** We choose the infinity large plane-xy, which enclose the upper hemisphere. For we have already got

For ,

,

🡪

**Total force**  +

* **Method 3.** Inside the bulk

 and

🡪

1. **Angular momentum of field**

The energy density stored in the field:

**Momentum density**: ( --CGS )

We define the angular momentum density:

Force density:

Torque density: 🡪 Levi-civita symbol

**Define:**

🡪 (, since )

**angular momentum density angular momentum density flow torque density transformed from mechanical degree of freedom.**

* **Example** Ex. 8.4 Page 371 Griffiths’ book

****Consider a very long solenoid with radius , turns per unit length, and current . Inside and outside are two charged cylindrical shells with and , respectively. When the current in the solenoid turn off gradually, we know that the cylinders begin to rotate. Where does the angular momentum come from?

**Solution:**

1. The -field inside the solenoid induced by changing is

 🡪

For , Faraday’s Law

🡪

For ,

🡪

 is along the tangential direction 🡪 The toque on the **outer** cylinder () is

The toque on the **inner** cylinder () is

🡪 The mechanical angular momenta together .

Now let us check the angular momentum density from the field.

 for (using Gauss’s Law)

-field only lies inside the solenoid at , in which

🡪 for

This amount of angular momentum is transferred to mechanical angular momentum.